LESSON

12.1 Probability

Florida Standards

The student is expected to:

FL Statistics and Probability—7.SP.3.5

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.

FL Statistics and Probability—7.SP.3.7a

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

Mathematical Practices

FL MP.6.1 Precision

Engage

ESSENTIAL QUESTION

How can you describe the likelihood of an event? Sample answer: I can describe the likelihood as certain, likely, as likely as not, unlikely, or impossible.

Motivate the Lesson

Ask: How can you decide the likelihood that you will win in a game of chance? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY

Focus on Reasoning

Encourage students to list the possible numbers that correspond to each event. Discuss how and why they can compare the numbers that correspond to each event to help order the events.

Explain

EXAMPLE 1

Connect Vocabulary ELL

Check that students understand the difference between an *outcome* and an *event*. In the experiment of rolling a six-sided number cube, rolling a 1 is both a possible outcome and an event. Rolling a 1 or a 2 is a different event that includes two possible outcomes, 1 and 2.

Questioning Strategies **Mathematical Practices**

- The event for the experiment in A is certain. What is an event that is impossible? Justify your answer. Rolling any number greater than 6 is impossible since any number greater than 6 is not a possible outcome.
- How can you find the possible outcomes to support the claim in B, that the sum of the numbers is 3 when you roll two number cubes is unlikely? What are the possible sums? Sample answer: Make an organized list to show all the sums possible when rolling two number cubes. Possible sums: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
- How do you know that both the events in **C** and **D** are as likely as not? If there are as many ways for an event to occur as not to occur, then the event is as likely as not. For C, 2, 4, 6, 8, and 10 are even while 1, 3, 5, 7, and 9 are odd. For D, 2, 3, 5, and 7 are prime while 0, 1, 4, and 6 are not prime.

ADDITIONAL EXAMPLE 1

Tell whether each event is impossible, unlikely, as likely as not, likely, or certain. Then tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1.

A You roll a six-sided number cube and the number is less than 2. unlikely; close to 0

B You roll two number cubes and the sum of the numbers is 1. impossible; 0

C A bag contains 3 blue marbles and 3 red marbles. You select a red marble from the bag at random. as likely as not; $\frac{1}{2}$

D A spinner has 5 equal sections marked 1 through 5. You spin and land on a number less than 5. likely; close to 1

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LESSON

12.1 Probability

FL 7.SP.3.5

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. . . . *Also 7.SP.3.7a*

ESSENTIAL QUESTION

How can you describe the likelihood of an event?

EXPLORE ACTIVITY FL 7.SP.3.5

Finding the Likelihood of an Event

Each time you roll a number cube, a number from 1 to 6 lands face up. This is called an *event*.

Work with a partner to decide how many of the six possible results of rolling a number cube match the described event.

Then order the events from least likely (1) to most likely (9) by writing a number in each box to the right.

Rolling a number less than 7	1, 2, 3, 4, 5, 6; 6 of 6 possible rolls	9
Rolling an 8	0 of 6 possible rolls	1
Rolling a number greater than 4	5, 6; 2 of 6 possible rolls	3/4
Rolling a 5	5; 1 of 6 possible rolls	2
Rolling a number other than 6	1, 2, 3, 4, 5; 5 of 6 possible rolls	8
Rolling an even number	2, 4, 6; 3 of 6 possible rolls	5/6
Rolling a number less than 5	1, 2, 3, 4; 4 of 6 possible rolls	7
Rolling an odd number	1, 3, 5; 3 of 6 possible rolls	5/6
Rolling a number divisible by 3	3, 6; 2 of 6 possible rolls	3/4

Reflect

1. Are any of the events impossible? Sample answer: Rolling an 8 is impossible because there is no 8 on the number cube.

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Describing Events

An **experiment** is an activity involving chance in which results are observed. Each observation of an experiment is a **trial**, and each result is an **outcome**. A set of one or more outcomes is an **event**.

The **probability** of an event, written P(event), measures the likelihood that the event will occur. Probability is a measure between 0 and 1 as shown on the number line, and can be written as a fraction, a decimal, or a percent.

If the event is not likely to occur, the probability of the event is close to 0. If an event is likely to occur, the event's probability is closer to 1.

Impossible	Unlikely	As likely as not	Likely	Certain
0		$\frac{1}{2}$		1
0		0.5		1.0
0%		50%		100%

EXAMPLE 1 Real World FL 7.SP.3.5

Tell whether each event is impossible, unlikely, as likely as not, likely, or certain. Then, tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1.

A You roll a six-sided number cube and the number is 1 or greater.

This event is certain to happen. Its probability is 1.

Because you can roll the numbers 1, 2, 3, 4, 5, and 6 on a number cube, there are 6 possible outcomes.

B You roll two number cubes and the sum of the numbers is 3.

This event is unlikely to happen. Its probability is close to 0.

C A bowl contains disks marked with the numbers 1 through 10. You close your eyes and select a disk at random. You pick an odd number.

This event is as likely as not. The probability is $\frac{1}{2}$.

D A spinner has 8 equal sections marked 0 through 7. You spin and land on a prime number.

This event is as likely as not. The probability is $\frac{1}{2}$.

Remember that a prime number is a whole number greater than 1 and has exactly 2 divisors, 1 and itself.

Math Talk Mathematical Practices

Is an event that is *not* certain an impossible event? Explain.

Sample answer: No; it is unlikely that I will draw a blue marble at random out of a jar containing 35 yellow marbles and one blue one, but it could happen.

Reflect

2. The probability of event A is $\frac{1}{3}$. The probability of event B is $\frac{1}{4}$. What can you conclude about the two events?

Neither is very likely, but event A is more likely to happen than event B, because $\frac{1}{3} > \frac{1}{4}$.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.6.1

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to display, explain, and justify mathematical ideas … using precise mathematical language in written or oral communication. Students learn the definitions for probabilistic events and connect the likelihood of an event to probabilities. Next, they identify the sample space for an event and use a ratio to find the probability of a simple event. Finally, students find the complement of an event. In this way, students are able to use precise language to communicate about probability.

Math Background

Considering the complement of an event can sometimes offer a shortcut when calculating theoretical probabilities. Given an event E, the complement of the event (denoted $\overline{E}$) is the set of all outcomes not included in the event. An event and its complement are disjoint and together form the entire sample space. Hence, $P(E) + P(\overline{E}) = 1$ and so $P(E) = 1 - P(\overline{E})$. The difference is often useful in calculating probabilities of events that contain many outcomes. In such cases, it may be easier to calculate the probability of the complement and then subtract this value from 1.

YOUR TURN

Talk About It

Check for Understanding

Ask: For the hat experiment, what is an event that is likely? unlikely? How do you know? Sample answer: A likely event is picking a number greater than 3 because there are more numbers from 4 to 16 than from 1 to 3. An unlikely event is picking a number less than 3 because there are fewer numbers less than 3 than from 3 to 16.

ADDITIONAL EXAMPLE 2

What is the probability of rolling a multiple of 3 on a standard number cube? $\frac{1}{3}$

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EXAMPLE 2

Questioning Strategies — Mathematical Practices

- Why do you find the sample space for an experiment to find the probability of an event? The sample space lists the total number of possible outcomes for an experiment. This is the denominator of the probability ratio.
- How do you decide which outcomes are the favorable outcomes? The favorable outcomes are the outcomes that correspond to the event.

Engage with the Whiteboard

Invite a student volunteer to rewrite the probabilty of rollling an even number as a decimal and as a percent.

Talk About It

Check for Understanding

Ask: How does the probability of rolling an odd number compare to the probability of rolling an even number? Explain. It is the same. There are still 3 favorable outcomes (1, 3, and 5) and 6 possible outcomes.

Avoid Common Errors

Students may write the probability ratio with the number of unfavorable outcomes in the denominator. Remind them that the total number of possible outcomes should go in the denominator.

YOUR TURN

Focus on Modeling — Mathematical Practices

Encourage students to list the sample spaces involved in Problems 4 and 5. This will help them in setting up the probability ratios that represents each event.

Integrating the ELPS — ELL

Encourage English learners to take notes on new terms or concepts and to write them in familiar language.

ADDITIONAL EXAMPLE 3

There are 4 aces in a standard deck of 52 cards. What is the probability of not getting an ace if you select one card at random? $\frac{12}{13}$

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EXAMPLE 3

Questioning Strategies — Mathematical Practices

- The probability of choosing a card with an even number at random from a deck of 52 cards is $\frac{20}{52}$ or $\frac{5}{13}$. Can you use the probability of the complement to find the probability of choosing a card with an odd number at random? Justify your answer. No, the complement includes both the cards with odd numbers and the face cards.

Engage with the Whiteboard

Ask a volunteer to first simplify the probability of getting a red jack. Then have the student use the formula to verify that the probability of the complement is the same.

Focus on Reasoning — Mathematical Practices

Connect the likelihood of an event to its complement. Discuss why events that are impossible have complements that are certain, events that are unlikely have complements that are likely, and so on.

YOUR TURN

3. A hat contains pieces of paper marked with the numbers 1 through 16. Tell whether picking an even number is impossible, unlikely, as likely as not, likely, or certain. Tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1.
as likely as not; $\frac{1}{2}$

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Finding Probability

The **sample space** is a set of all possible outcomes for an event. A sample space can be small, such as the 2 outcomes when a coin is flipped. Or a sample space can be large, such as the possible number of Texas Classic automobile license plates. Identifying the sample space can help you calculate the probability of an event.

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Probability of An Event

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of equally likely possible outcomes}}$$

EXAMPLE 2 Real World FL 7.SP.3.7a

What is the probability of rolling an even number on a standard number cube?

STEP 1 Find the sample space for a standard number cube.

{1, 2, 3, 4, 5, 6} *There are 6 possible outcomes.*

STEP 2 Find the number of ways to roll an even number.

2, 4, 6 *The event can occur 3 ways.*

STEP 3 Find the probability of rolling an even number.

$$P(\text{even}) = \frac{\text{number of ways to roll an even number}}{\text{number of faces on a number cube}}$$

$$= \frac{3}{6} = \frac{1}{2}$$ *Substitute values and simplify.*

The probability of rolling an even number is $\frac{1}{2}$.

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YOUR TURN

Find each probability. Write your answer in simplest form.

4. Picking a purple marble from a jar with 10 green and 10 purple marbles. $\frac{1}{2}$

5. Rolling a number greater than 4 on a standard number cube. $\frac{1}{3}$

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Using the Complement of an Event

The **complement** of an event is the set of all outcomes in the sample space that are *not* included in the event. For example, in the event of rolling a 3 on a number cube, the complement is rolling any number other than 3, which means the complement is rolling a 1, 2, 4, 5, or 6.

An Event and Its Complement

The sum of the probabilities of an event and its complement equals 1.

$$P(\text{event}) + P(\text{complement}) = 1$$

You can apply probabilities to situations involving random selection, such as drawing a card out of a shuffled deck or pulling a marble out of a closed bag.

EXAMPLE 3 Real World FL 7.SP.3.7a

There are 2 red jacks in a standard deck of 52 cards. What is the probability of not getting a red jack if you select one card at random?

$P(\text{event}) + P(\text{complement}) = 1$

$P(\text{red jack}) + P(\text{not a red jack}) = 1$ *The probability of getting a red jack is $\frac{2}{52}$.*

$\frac{2}{52} + P(\text{not a red jack}) = 1$ *Substitute $\frac{2}{52}$ for P(red jack).*

$\frac{2}{52} + P(\text{not a red jack}) = \frac{52}{52}$ *Subtract $\frac{2}{52}$ from both sides.*

$-\frac{2}{52} \qquad -\frac{2}{52}$

$P(\text{not a red jack}) = \frac{50}{52}$

$P(\text{not a red jack}) = \frac{25}{26}$ *Simplify.*

The probability that you will not draw a red jack is $\frac{25}{26}$. It is likely that you will not select a red jack.

DIFFERENTIATE INSTRUCTION

Communicating Math

Have students suggest events that are impossible, unlikely, as likely as not, likely, or certain. Do all students agree with the likelihoods of the suggested events? Discuss the reasons for any differences. When possible, work with students to find the probabilities of their suggested simple events. Contrast the precision of the numerical probability with the intuitive likelihood description of the event.

Home Connection

Encourage students to play a game at home with family members and then to write responses to the following questions on notebook paper. Discuss students' answers in class.

What game did you play?

Before starting the game, did every player have an equal chance or likelihood of winning the game? How do you know?

How do skill and strategy affect the outcome of the game?

What other factors affect the outcome of the game?

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

YOUR TURN

Focus on Communication **Mathematical Practices**

Discuss the advantages of using the rule for the complement to find each probability.

Elaborate

Talk About It

Summarize the Lesson

Ask: How are the probability and the likelihood of an event related? If the probability is 0, the event is impossible. If the probability is close to 0, the event is unlikely. If the probability is $\frac{1}{2}$, the event is as likely as not. If the probability is close to 1, the event is likely. If the probability is 1, the event is certain.

GUIDED PRACTICE

Engage with the Whiteboard

Have students list the numbers that correspond to each event in Exercises 2–5.

Avoid Common Errors

Exercise 6 Students are used to even numbers having a probability of $\frac{1}{2}$ in other contexts and may give that incorrect answer here. Remind them to read each item carefully.

Exercises 8–11 Remind students to use the rule for the complement to find each probability.

Reflect

6. Why do the probability of an event and the probability of its complement add up to 1?

The complement is made up of all outcomes not in the event. When you put the outcomes of an event and its complement together, you get all possible outcomes of an event. The probability of getting all the possible outcomes equals 1.

YOUR TURN

7. A jar contains 8 marbles marked with the numbers 1 through 8. You pick a marble at random. What is the probability of not picking the marble marked with the number 5? $\frac{7}{8}$

8. You roll a standard number cube. Use the probability of rolling an even number to find the probability of rolling an odd number. $\frac{1}{2}$

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Guided Practice

1. In a hat, you have index cards with the numbers 1 through 10 written on them. Order the events from least likely to happen (1) to most likely to happen (8) when you pick one card at random. In the boxes, write a number from 1 to 8 to order the eight different events. (Explore Activity)

You pick a number greater than 0.	8
You pick an even number.	5
You pick a number that is at least 2.	7
You pick a number that is at most 0.	1
You pick a number divisible by 3.	3
You pick a number divisible by 5.	2
You pick a prime number.	4
You pick a number less than the greatest prime number.	6

Guided Practice

Determine whether each event is impossible, unlikely, as likely as not, likely, or certain. Then, tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1. (Example 1)

2. randomly picking a green card from a standard deck of playing cards
impossible; 0

3. randomly picking a red card from a standard deck of playing cards
as likely as not; $\frac{1}{2}$

4. picking a number less than 15 from a jar with papers labeled from 1 to 12
certain; 1

5. picking a number that is divisible by 5 from a jar with papers labeled from 1 to 12
unlikely; close to 0

Find each probability. Write your answer in simplest form. (Example 2)

6. spinning a spinner that has 5 equal sections marked 1 through 5 and landing on an even number
$\frac{2}{5}$

7. picking a diamond from a standard deck of playing cards which has 13 cards in each of four suits: spades, hearts, diamonds and clubs
$\frac{1}{4}$

Use the complement to find each probability. (Example 3)

8. What is the probability of not rolling a 5 on a standard number cube?
$\frac{5}{6}$

9. A spinner has 3 equal sections that are red, white, and blue. What is the probability of not landing on blue? $\frac{2}{3}$

10. A spinner has 5 equal sections marked 1 through 5. What is the probability of not landing on 4?
$\frac{4}{5}$

11. There are 4 queens in a standard deck of 52 cards. You pick one card at random. What is the probability of not picking a queen?
$\frac{12}{13}$

ESSENTIAL QUESTION CHECK-IN

12. Describe an event that has a probability of 0% and an event that has a probability of 100%.

Sample answer: pulling a red marble out of a bag that contains only blue marbles; pulling a white marble out of a bag that contains only white marbles.

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Online homework assignment available

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12.1 LESSON QUIZ

FL 7.SP.3.5

Tell whether each event is impossible, unlikely, as likely as not, likely, or certain. Then tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1.

1. rolling a number less than 4 on a standard number cube
2. randomly picking a king from a standard deck of playing cards

You pick one tile without looking from a bag that has 2 blue, 3 red, and 5 yellow tiles. Find each probability. Write your answer in simplest form.

3. picking a blue tile
4. picking a yellow tile
5. not picking a red tile

Lesson Quiz available online

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Answers

1. as likely as not; $\frac{1}{2}$
2. unlikely; close to 0
3. $\frac{1}{5}$
4. $\frac{1}{2}$
5. $\frac{7}{10}$

Evaluate

GUIDED AND INDEPENDENT PRACTICE

FL 7.SP.3.5, 7.SP.3.7a

Concepts & Skills	Practice
Explore Activity Finding the Likelihood of an Event	Exercises 1, 16
Example 1 Describing Events	Exercises 2–5, 15, 18
Example 2 Finding Probability	Exercises 6–7, 13, 17, 19
Example 3 Using the Complement of an Event	Exercises 8–11, 14, 18

Exercise	Depth of Knowledge (D.O.K.)	FL Mathematical Practices
13	**2** Skills/Concepts	**MP.4.1** Problem Solving
14	**3** Strategic Thinking H.O.T.	**MP.4.1** Problem Solving
15	**3** Strategic Thinking H.O.T.	**MP.4.1** Problem Solving
16	**3** Strategic Thinking H.O.T.	**MP.6.1** Precision
17	**3** Strategic Thinking H.O.T.	**MP.4.1** Problem Solving
18	**3** Strategic Thinking H.O.T.	**MP.6.1** Precision
19	**2** Skills/Concepts	**MP.8.1** Patterns
20	**3** Strategic Thinking H.O.T.	**MP.8.1** Patterns
21	**3** Strategic Thinking H.O.T.	**MP.8.1** Patterns
22	**3** Strategic Thinking H.O.T.	**MP.6.1** Precision

Additional Resources

Differentiated Instruction includes:

- Leveled Practice Worksheets

Name _____ Class _____ Date _____

12.1 Independent Practice

FL 7.SP.3.5, 7.SP.3.7a

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13. There are 4 aces and 4 kings in a standard deck of 52 cards. You pick one card at random. What is the probability of selecting an ace or a king? Explain your reasoning.
$\frac{2}{13}$; The event can occur in 8 ways. There are 52 outcomes in the sample space. $\frac{8}{52} = \frac{2}{13}$.

14. There are 12 pieces of fruit in a bowl. Seven of the pieces are apples and two are peaches. What is the probability that a randomly selected piece of fruit will not be an apple or a peach? Justify your answer.
$\frac{1}{4}$; Since 7 pieces of fruit are apples and 2 are peaches, 3 pieces are not apples or peaches. $P(\text{not an apple or a peach}) = \frac{3}{12} = \frac{1}{4}$.

15. **Critique Reasoning** For breakfast, Clarissa can choose from oatmeal, cereal, French toast, or scrambled eggs. She thinks that if she selects a breakfast at random, it is likely that it will be oatmeal. Is she correct? Explain your reasoning.
No, it is unlikely that she will have oatmeal for breakfast. Since there are 4 choices, the probability that she will choose oatmeal is $\frac{1}{4}$, or 25%.

16. **Draw Conclusions** A researcher's garden contains 90 sweet pea plants, which have either white or purple flowers. About 70 of the plants have purple flowers, and about 20 have white flowers. Would you expect that one plant randomly selected from the garden will have purple or white flowers? Explain.

purple; There are a lot more plants with purple flowers than with white flowers. The probability of selecting a white-flowered plant is $\frac{2}{9}$, while the probability of selecting a purple-flowered plant is $\frac{7}{9}$.

17. The power goes out as Sandra is trying to get dressed. If she has 4 white T-shirts and 10 colored T-shirts in her drawer, is it likely that she will pick a colored T-shirt in the dark? What is the probability she will pick a colored T-shirt? Explain your answers.
Because she has more colored T-shirts than white T-shirts, it is likely that she will pick a colored T-shirt; $\frac{\text{colored T-shirts}}{\text{total T-shirts}} = \frac{10}{10+4} = \frac{5}{7}$

18. James counts the hair colors of the 22 people in his class, including himself. He finds that there are 4 people with blonde hair, 8 people with brown hair, and 10 people with black hair. What is the probability that a randomly chosen student in the class does not have red hair? Explain.
1; None of the students in the class have red hair, so it is certain that a randomly chosen student will not have red hair.

19. **Persevere in Problem Solving** A bag contains 8 blue coins and 6 red coins. A coin is removed at random and replaced by three of the other color.

a. What is the probability that the removed coin is blue?
$\frac{8}{14} = \frac{4}{7}$

b. If the coin removed is blue, what is the probability of drawing a red coin after three red coins are put in the bag to replace the blue one?
$8 - 1 = 7$ blue coins and $6 + 3 = 9$ red coins; $\frac{9}{16}$

c. If the coin removed is red, what is the probability of drawing a red coin after three blue coins are put in the bag to replace the red one?
$8 + 3 = 11$ blue coins and $6 - 1 = 5$ red coins; $\frac{5}{16}$

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

20. **Draw Conclusions** Give an example of an event in which all of the outcomes are not equally likely. Explain.
Sample answer: If some marbles in a jar are heavier than others, then the heavier marbles would sink and be less likely to be selected.

21. **Critique Reasoning** A box contains 150 black pens and 50 red pens. Jose said the sum of the probability that a randomly selected pen will not be black and the probability that the pen will not be red is 1. Explain whether you agree.
Yes; Because not selecting red means black will be selected, and vice versa. So, $P(\text{not black}) + P(\text{black}) = P(\text{not black}) + P(\text{not red}) = 1$.

22. **Communicate Mathematical Ideas** A spinner has 7 identical sections. Two sections are blue, 1 is red, and 4 of the sections are green. Suppose the probability of an event happening is $\frac{2}{7}$. What does each number in the ratio represent? What outcome matches this probability?
2 is the number of ways the event can happen; 7 is the number of outcomes in the sample space; landing on blue

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

A sample space consists of 26 separate events. Each event is equally likely.

a. What is the probability of each event? $\frac{1}{26}$

b. What is the probability of the complement of each event? $\frac{25}{26}$

c. Is the probability of each event highly likely or highly unlikely? Explain. The probability is highly unlikely because $\frac{1}{26}$ is close to 0.

d. Describe a sample space that consists of 26 separate events. Sample answer: the 26 letters of the English alphabet

e. For the sample space, describe an experiment with an event that is as likely as not. Sample answer: Draw, at random, any letter from A through M from a hat containing the 26 letters of the alphabet.

f. Write and answer a different probability question based on your sample space. Sample answer: What is the probability of drawing a vowel from a hat that contains every letter of the alphabet? $\frac{5}{26}$